Conditional Truth Table and Validity Problem: Discussion

Assignment: Translate the following argument, and then build a truth table to decide if the argument is ***valid*** or ***invalid***

Argument:

1. If it's not sunny, then we're not having a picnic.
2. We are having a picnic.

(so) It's sunny.

(**P**: It's sunny; **Q**: We are having a picnic.)

Discussion: We first translate the argument into formal language. The first premise contains three form phrases: "if… then," and two cases of "not".

If it's **not** sunny, **then** we're **not** having a picnic

What's left beyond these form phrases are the subject matter sentences "It's sunny" and "We're having a picnic". Replacing subject matter sentences with sentence letters, we have the following.

If not P, then not Q.

Since "then" occurs right by the comma, we conclude that the conditional phrase "if… then" is the main connective – with "if" before the antecedent, and "then" before the consequent. "If… then" is translated by the arrow.

(not P $\rightarrow$ not Q)

Each "not" is translated by a tilde, completing the translation of the first premise.

1. (~P $\rightarrow$ ~Q)

The second premise is "We're having a picnic." Since our translation table already assigns this sentence the sentence letter "Q," we translate the second premise as "Q".

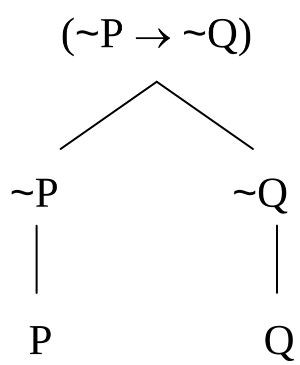
1. (~P → ~Q)
2. Q

The conclusion is "It's sunny." Since our translation table already assigns this sentence the sentence letter "P," we translate the conclusion as "P".

1. (~P → ~Q)
2. Q

∴ P

The first premise is the only sentence here larger than a sentence letter. Its construction tree is as follows.



That means the truth table steps for the first premise will be as follows.

P	**Q**	**~P**	**~Q**	**(~P → ~Q)**
1	1			
1	0			
0	1			
0	0			

Using the Negation Rule, we build truth tables for the antecedent, “~P,” and consequent, “~Q”.

●	~●
1	0
0	1

		Antecedent	Consequent	
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**
1	1	0	0	
1	0	0	1	
0	1	1	0	
0	0	1	1	

The truth table for the whole conditional is built from the truth tables of its parts, following the Conditional Rule.

In the first valuation, the antecedent is false and the consequent is false. In this sort of valuation, the Conditional Rule says the whole conditional is **true**.

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	1
0	**0**	**1**

		Antecedent	Consequent	
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**
1	1	**0**	**0**	**1**
1	0	0	1	
0	1	1	0	
0	0	1	1	

In the second valuation, the antecedent is false, but the consequent is true. This is the sort of valuation that makes a conditional **true**.

●	▲	(● → ▲)
1	1	1
1	0	0
0	**1**	**1**
0	0	1

		Antecedent	**Consequent**	
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**
1	1	0	0	1
1	0	**0**	**1**	**1**
0	1	1	0	
0	0	1	1	

In the third valuation, the antecedent is true, but the consequent is false. The Conditional Rule says a conditional is **false** in this sort of valuation.

●	▲	(● → ▲)
1	1	1
1	**0**	**0**
0	1	1
0	0	1

		Antecedent	**Consequent**	
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**
1	1	0	0	1
1	0	0	1	1
0	1	**1**	**0**	**0**
0	0	1	1	

The last valuation makes both the antecedent and the consequent true. According to the Conditional Rule, a conditional is **true** in this sort of valuation.

●	▲	(● → ▲)
1	**1**	**1**
1	0	0
0	1	1
0	0	1

		Antecedent	**Consequent**	
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**
1	1	0	0	1
1	0	0	1	1
0	1	1	0	0
0	**0**	**1**	**1**	**1**

The second premise of the argument is "Q," which already shows up in the truth table. Likewise, the conclusion is "P" which is also already present in the truth table. So the truth table for the entire argument is complete. (I copy the conclusion, "P", to the right end of the table, just so I can read the truth table from left to right.)

	Premise (2)			**Premise (1)**	∴
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**	**P**
1	1	0	0	1	1
1	0	0	1	1	1
0	1	1	0	0	0
0	0	1	1	1	0

Our truth table test of validity remains unchanged from the previous chapter. First we pick out the valuations where all of the premises are true at the same time. Here, there is only one valuation which makes all the premises true: Valuation (1).

	Premise (2)			**Premise (1)**	**∴**
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**	**P**
1	**1**	0	0	**1**	1
1	0	0	1	1	1
0	1	1	0	0	0
0	0	1	1	1	0

Next, we see whether the conclusion whenever the premises are.

Since there was only one valuation making all of the premises true – Valuation (1) – that is the only ‘selected’ valuation. And in Valuation (1) the conclusion is true as well.

	Premise (2)			**Premise (1)**	**∴**
P	**Q**	**~P**	**~Q**	**(~P → ~Q)**	**P**
1	**1**	0	0	**1**	**1**
1	0	0	1	1	1
0	1	1	0	0	0
0	0	1	1	1	0

Every time the premises are all true, the conclusion is also true. In other words: this argument is **valid**.